Human Capital Formation and Quasi-Hyperbolic Discounting

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March 20, 2021

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Abstract

This paper examines the implications of populating a model of human capital formation with both rational and present-biased agents. We first introduce analytical results that outline the distinctions that arise between rational exponential discounting (RE) and quasi-hyperbolic discounting (QH) agents in equilibrium and find QH agents invest less in human capital and have lower retirement consumption that RE agents. We then examine several policy interventions and their impact on human capital formation, consumption, and utility. We find that dedicating tax revenue to funding an education incentives program increases human capital attainment and lifetime consumption for both RE and QH agents relative to exclusively funding pay-as-you-go social security *and* relative to a government regime in which no taxes are levied. Further, education incentive funding reduces the gap between expected and realized lifetime for QH households compared to both an environment in which taxes are exclusively dedicated to funding social security and an environment in which no taxes are levied.

1. Introduction

In the United States and many other developed countries, the decision to attend school beyond the state (or nationally) mandated age rests in the hands of young individuals. The level of human capital investment chosen by these young individuals will play a large role in determining their lifetime income, feasible consumption, and wealth. We contribute to the literature examining the implications of choosing non-mandatory education when young, particularly when young agents may suffer from behavioral biases through the study non-mandatory schooling choices in societies populated by rational exponential (RE) and quasi-hyperbolic discounters (QH).

Using a three period overlapping generations (OLG) model, we provide several contributions to the literature. First, we show analytically that QH agents will invest less in education and have lower retirement consumption than RE agents. Although a growing empirical literature has connected present-biased preferences to lower school attainment, this is the first paper that corroborates this results analytically. Second, we investigate how varying the allocation of tax revenue between education subsidies and social security changes the investment decision and lifetime consumption of both QH and RE agents. We find increasing the proportion of tax revenue dedicated to education subsidies leads to increased consumption in all periods for both QH and RE agents. Finally, we show that education subsidies can both minimize the gap between expected and realized utility for QH households **and** be welfare enhancing from the perspective of a retired QH household relative to an environment in which no taxes are levied.

Human capital accumulation has been integrated into a number of macroeconomic models in order to better understand the role of skill upgrading on economic activity. However, much of the early literature on this topic speaks to the optimizing decisions made by outside agents (parents and government) in the human capital of the young rather than focusing on young agents who must decide how much to consume and save while simultaneously deciding to invest time in furthering their education (Galor and Zeira, 1993; Glomm and Ravikumar, 1992; Zhang, 1995). A clear exception is Ben-Porath (1967), however the continuous investment in human capital at the center of the Ben-Porath model is hard to remedy with our modern education system in which human capital is acquired via formal college training and not continual on-the-job training.

We focus on the investment decision of young agents in order to understand how disutility from education crowds out human capital investments. This tension between current utility and future labor market returns is particularly interesting in the presence of present biased agents who inherently overweight immediate returns over future payoffs. We contribute to a growing literature interested in rationalizing reduced human capital investment associated with present bias and a distinct literature outlining the trade-off between government outlays dedicated to funding social security and public education.

The remainder of our paper is structured as follows. Section 2 provides an overview of related literature. Section 3 is dedicated to outlining some general features of our model and the corresponding characterization of an equilibrium. Section 4 delineates equilibrium differences that arise between QH and RE agents under different policy regimes governing the dispersion of tax revenue. Section 5 concludes.

2. Related Literature

We contribut to a growing literature modeling the trade off between government outlays dedicated to funding social security and public education. Pecchenino and Pollard (2002) and Kaganovich and Zilcha (1999) analyze this trade off utilizing a 2 period OLG model in which all educational investment is made by parents and the government. Kaganovich and Zilcha find funding for social security is welfare and growth reducing relative to equivalent funding allocated to education. Pecchenino and Pollard corroborate this finding even in a model that does not assume crowding out in spending and allows for societal aging. Our policy analysis leads to the same conclusions regarding the negative impact of social security for both RE and QH societies. Annabi et al. (2011) also find that higher education incentives can increase human capital and offset the effects of declining labor force growth. Glomm and Kaganovitch (2008) analyze the trade off between funding social security and public education in a model with heterogeneous agents. They find that social security funding reduces income inequality and, under certain circumstances, does not decrease growth.

Several recent exceptions to modeling youth education from the perspective of parents and the government are Maneulli and Seshadri (2014), Kruger and Ludwig (2016), and Cowan (2016). Kruger and Ludwig characterize the optimal mix of income taxes and education subsidies in an OLG model and find the welfare-maximizing policy includes larger tax progressivity and a greater education subsidy than currently offered under US policy. We come to a similar conclusion in our model, despite the fact that our modeling environments have considerable differences and our focus is on the distinctions that arise between QH and RE societies. Cowan uses a simple model of education choice to estimate the impact of heterogeneous exponential discount rates on human capital accumulation, concluding that credit constraints play a key role in generating differences in human capital attainment. Unlike Cowan, we focus on short-term discount rate heterogeneity and find large distinctions in schooling between QH and RE agents even in a modeling environment with no credit constraints.

We also contribute to the literature examining the theoretical and empirical effect of quasi-hyperbolic discounting in economics. Quasi-hyperbolic discounting has a long history of integration in the scientific literature. First introduced by Strotz (1956) and Phelps and Pollak (1968) in the middle of the 20th century, quasi-hyperbolic discounting gained significant traction in the economics literature with its reintroduction by Laibson (1997). Laibson found that QH discounters would overspend and under-save if given the opportunity. These insights have been applied to education in several empirical studies, with Cadena and Keys (2015), DePaola and Scoppa (2015), and DePaola and Gioia (2017) all finding evidence that present biased individuals are less likely to complete college and more likely to express regret in middle aged. Our model marks the first attempt to rationalize these empirical findings in a theoretical model.

In the context of our model, optimizing agents are forced to commit to a level of human capital investment in the first period of their lives conditional on their expected savings and consumption profile in the second period of their lives. However, QH optimizers will choose to re-allocate savings to consumption when the second period arrives leaving themselves with much lower savings to consume out of when old and less human capital (and thus middle aged wages) than RE agents. Banks et al. (1998) find significant evidence that consumption falls when households retire and they conclude "the systematic arrival of unexpected adverse information" is the most likely culprit for this observation. Our 3-period model in which agents face no aggregate or idiosyncratic uncertainty generates this same qualitative prediction. That is, when agents are present biased, under-saving for retirement is a natural result of optimizing behavior. This finding compliments recent empirical work by Huffman et al. (2016) and Schreiber and Weber (2017), who point to impatience as a likely pathway through which retirement wealth, and thus consumption, is lowered.

3. Modeling Overview

We assume that there are an infinite number of periods populated by overlapping generations of three-period-lived agents. The subscript t denotes a period in time and a superscript t denotes the generation born at time t (e.g. the period $t+2$ consumption of an agent born in period t will be written as c^t_{t+2}). Each generation consists of a continuum of agents and the size of each generation is normalized to unity. There is a single good (c) that is produced, consumed, and saved at the rate r. We assume that agents live in a small, open economy so that r is exogenously determined and fixed. Credit access is limited only by the present discounted value of one's lifetime income.

Young agents are endowed with one unit of time and are faced with a tradeoff between spending time working and receiving a wage w^t_t and attending non-mandatory schooling ($s^t_t \in [0, 1]$) to increase their human capital when middle aged (h^t_{t+1}). We assume that agents receive disutility from their schooling investment. Without this assumption, agents in both RE and QH societies would chose the same level of schooling; that which maximizes their lifetime income[1]. Therefore, we proceed as in Cowan (2016) by imposing disutility from education. When young, agents can consume out of their exogenous endowment (x), using their wage earnings, or by financing their consumption through borrowing at the rate r. If they do not consume their entire income (the sum of wage earnings and the exogenous endowment), agents can store their savings (a_t^t) at the rate r. Middle age agents are endowed with one unit of time which they supply inelastically working for a wage of w_{t+1}^t, which is an increasing function of their human capital. When old, agents do not work, and consume whatever they have saved in the previous period as well as the interest accrued on their savings and any social security transfer from the government.

We interpret the schooling variable in the following way: $s = 0$ corresponds to the education of an individual who drops out after 10th grade and $s = 1$ corresponds to an individual who receives 4 years of education beyond a Bachelor's degree. Every one tenth increase in s maps to a 1 year increase in education beyond 10th grade. Although the true wage premium is kinked at values of s corresponding to the completion of certain grade levels (i.e. high-school, Bachelor's degree, etc.), we abstract from this distinction in our simplified model and instead focus on continuous, differentiable functions of human capital accumulation.

Members of generation t derive utility V_t^t from consumption in all three periods of life and experience disutility from schooling (s_t^t) when young.[2] That is:

$$V_t = U(c_t^t, c_{t+1}^t, c_{t+2}^t, 1 - s_t^t)$$

$U(\cdot, \cdot, \cdot, \cdot)$ is strictly concave and increasing in all of its arguments. This utility function holds for all agents in all generations. A rational exponential (RE) discounter is distinguished from

[1]See Becker 1967 and Fuchs 1982 for further exposition on this topic.

[2]We have considered specifications in which agents also received disutility from labor and find quantitatively similar results to those presented in Section 4. We do not include labor disutility in our model specification as it results in a model that is no longer analytically tractable. Further, if we assume that labor is additively separable in the utility function (as is common in the literature), then the schooling decision is orthogonal to an agent's choice of labor hours. As our focus is establishing analytical distinctions in schooling between QH and RE societies we proceed with the value function presented above.

a quasi-hyperbolic (QH) discounter in the following way:

RE discounting of future utility:

$$V_t = U(c_t) + v(1 - s_t) + \sum_{i=1}^{2} \beta^i U(c_{t+i})$$

QH discounting of future utility:

$$V_t = U(c_t) + v(1 - s_t) + \delta \sum_{i=1}^{2} \beta^i U(c_{t+i})$$

where $\delta \in (0, 1)$[3]. A QH agent applies an additional discount factor δ to all future utility that an RE agent does not apply to future utility.

In the following section, we proceed with $U(c) = \ln(c)$ in order to explicate equilibrium distinctions that arise between our two types of agents. The role of the government is the only distinction between the following three models, therefore the discussion of the government is left for each section. An equilibrium is defined by a collection of consumption and education decisions in which each generation solves their consumption and education profile according to the first order conditions implied by utility maximization and a government allocation of tax revenue that results in a balanced budget in each period.

3.1 Equilibrium Analysis: RE Solution

In this section we establish the equilibrium outcome for an RE society. The introduction of social security and education funding precludes analytical comparisons between RE and QH societies, thus in this section we assume that all government revenue that is collected via taxes on labor income is discarded so that the government budget is trivially balanced in each period.

[3]We have reversed the meaning of the β and δ parameters from Laibson's exposition of quasi-hyperbolic discounting so that the discount factor β retains its standard interpretation found throughout the macroeconomic literature.

Consider the following optimization problem solved by an RE agent born in period "t".

$$Max\ U_t = \ln(c_t^t) + \gamma \ln(1 - s_t^t) + \beta \ln(c_{t+1}^t) + \beta^2 \ln(c_{t+2}^t)$$

s.t.

$$\begin{aligned} c_t^t + a_t^t &= x + (1-\tau) w_t^t (1 - s_t^t) && (1) \\ c_{t+1}^t + a_{t+1}^t &= (1-\tau) w_{t+1}^t + (1+r) a_t^t && (2) \\ c_{t+2}^t &= (1+r) a_{t+1}^t && (3) \end{aligned}$$

Where $s_t^t \in [0,1]$ is the amount of schooling obtained by a young agent t, $x \geq 0$ is the exogenous initial endowment received by young agents of each generation, w_t is the wage paid to young, unskilled workers, $\gamma > 0$ is the weight applied to disutility from schooling, $\tau \in [0,1)$ is the exogenous tax rate, and $\beta \in (0,1)$ is the discount factor applied to future utility. Human capital evolves according to $h_{t+1}^t = (1 + \phi s_t^t)$ where $\phi > 0$. The production technology owned by young and middle age agents is of the form $f(h_t^t) = W_u h_t^t$. W_u can be thought of as the unskilled wage and $W_u \phi$ can be thought of as the skill premium. An agent born in period t produces $W_u(1 - s_t^t)$ units of the consumption good when young and $W_u(1 + \phi s_t^t)$ units of the consumption good when middle-aged. Agents face no uncertainty regarding their life span or their lifetime earnings and asset holdings.

As agents are identical, each generation will solve this problem in the same way.[4] Therefore, we express rational equilibrium consumption with subscripts y, m and o for young, middle aged, and old and the superscript R^*. That is, the equilibrium consumption of an RE agent in the first, second, or third period of her life is denoted by $c_y^{R^*}$, $c_m^{R^*}$, and $c_o^{R^*}$ respectively. The equilibrium schooling decision of an RE agent is denoted by s^{R^*}.

[4]See Appendix for an overview of analytical approach to solving the households problem.

$$
\begin{aligned}
c_y^{R^*} &= \frac{1}{1+\beta+\beta^2+\gamma}\left[x+\frac{(1-\tau)W_u(1+\phi)}{1+r}\right] &(4)\\
c_m^{R^*} &= \beta(1+r)c_y^{R^*} &(5)\\
c_o^{R^*} &= \beta^2(1+r)^2c_y^{R^*} &(6)\\
s^{R^*} &= 1-\frac{\gamma(1+r)}{(1-\tau)W_u[\phi-(1+r)]}c_y^{R^*} &(7)
\end{aligned}
$$

Table 1 outlines the partial derivatives associated with the equilibrium consumption and education profile of an optimizing rational agent.

Table 1: Baseline Model Partial Derivatives

	γ	x	β	r	W_u	ϕ	τ
$c_y^{R^*}$	-	+	-	-	+	+	-
s^{R^*}	-	-	+	-	+	+	-
$c_m^{R^*}$	-	+	+	+	+	+	-
$c_o^{R^*}$	-	+	+	+	+	+	-

These partial derivatives confirm basic intuition regarding the schooling decision of optimizing agents. Higher family wealth (x) lower schooling attainment due to disutility from schooling. A higher return to capital lowers schooling and first period consumption as agents invest in capital goods rather than human capital as r increases. Lower disutility from schooling (γ) or a higher return to schooling (ϕ) increase both education and consumption in all periods and more patient agents (higher β) will consume less when young but more when middle-aged and old through the combined effect of increased human capital (and thus earnings) and higher first period savings.

3.2 Equilibrium Analysis: QH Solution

We now consider an identical optimization problem but rather than assuming rational behavior, we populate our model with identical quasi-hyperbolic discounters. The resulting expected consumption profile for a QH agent (delineated by a Q^* superscript for actual equilibrium behavior and a Q' superscript for expected equilibrium behavior) who optimizes for her lifetime is:

$$\begin{aligned}
c_y^{Q^*} &= \frac{1}{1+\delta\beta+\delta\beta^2+\gamma}\left[x+\frac{(1-\tau)W_u(1+\phi)}{1+r}\right] && (8)\\
c_m^{Q'} &= \delta\beta(1+r)c_y^{Q^*} && (9)\\
c_o^{Q'} &= \delta\beta^2(1+r)^2 c_y^{Q^*} && (10)\\
s^{Q^*} &= 1-\frac{\gamma(1+r)}{(1-\tau)W_u[\phi-(1+r)]}c_y^{Q^*} && (11)
\end{aligned}$$

The above equilibrium for QH optimizers yields two immediate takeaways. First, for $\delta < 1$ QH agents consume more and invest less time to schooling when young than RE agents. Second, the planned middle aged and old age consumption profile for QH agents does *not* equal the actual consumption of middle aged and old QH agents. When our optimizing agents are rational, the path of consumption solved for by a young agent, the collection ($c_y^{R^*}$, $c_m^{R^*}$, and $c_o^{R^*}$), was the same as the actual consumption of young, middle-aged and old agents. But now with QH optimizers, $c_m^{Q'}$ and $c_o^{Q'}$ are no longer the true consumption profiles for middle-aged and old agents. That is, young agents incorrectly optimize for their future selves. To see why this is, we consider the utility of a QH agent from the perspective of being young, middle-aged and old:

$$\begin{aligned}
\tilde{U}_y &= \ln(\tilde{c}_y^t)+\gamma\ln(1-\tilde{s}^t)+\delta\beta\ln(\tilde{c}_m^t)+\delta\beta^2\ln(\tilde{c}_o^t)\\
\tilde{U}_m &= \beta^{-1}\ln(\tilde{c}_y^t)+\beta^{-1}\gamma\ln(1-\tilde{s}^t)+\ln(\tilde{c}_m^t)+\delta\beta\ln(\tilde{c}_o^t)\\
\tilde{U}_o &= \beta^{-2}\ln(\tilde{c}_y^t)+\beta^{-2}\gamma\ln(1-\tilde{s}^t)+\beta^{-1}\ln(\tilde{c}_m^t)+\ln(\tilde{c}_o^t)
\end{aligned}$$

When agents are middle-aged, they use a different discount rate to look back at decisions made when young (β) than they did when they were young looking forward to being middle-aged ($\beta\delta$). Thus, middle-aged agents must re-optimize and solve for their new optimal consumption and savings plan subject to the remainder of their lifetime budget constraint, taking the decisions of their young selves as given.[5]

With this interpretation in mind, we define $\tilde{a}_y^t$ as the savings of a generation t agent made in the first period of her life. Solving our agent's optimization problem and subbing in the optimal first period consumption, education and savings yields the following equilibrium consumption profile for middle-aged and old agents:

$$c_m^{Q^*} = \frac{1}{1+\beta\delta}\left[(1+r)a_y^{Q^*} + (1-\tau)W_u(1+\phi s^{Q^*})\right] \tag{12}$$

$$c_o^{Q^*} = \beta\delta(1+r)c_m^{Q^*} \tag{13}$$

Where $a_y^{Q^*} = x + (1-\tau)W_u(1-s^{Q^*}) - c_y^{Q^*}$. For QH agents, realized equilibrium consumption does not equal planned equilibrium consumption when middle-aged (or old). That is $c_m^{Q'} \neq c_m^{Q^*}$ and $c_o^{Q'} \neq c_o^{Q^*}$.

As we chose to represent our agent's utility as a natural log function, we are able to analytically compare the equilibrium consumption and schooling profile of rational exponential discounting agents relative to quasi-hyperbolic discounting agents. It is important to note that these differences are not unique to a log specification. Numerical analysis confirms these results for a general CRRA utility function. Table 2 contains the analytical equilibrium differences in schooling and consumption in each period of life between rational and quasi-hyperbolic agents. These differences hold for all values of τ between 0 and 1, all values of β and $\delta \in (0,1)$, all positive values of γ, and all non-negative values of x and W_u.

For the parameter space outlined above, RE agents consume less when young, consume more when old, and dedicate more time to schooling when young than QH agents. Middle-aged rational agents will consume more than middle-aged QH agents in equilibrium so long as $1 + \gamma > \delta(\beta^3 + \beta^2)$, otherwise QH agents will consume more than rational agents when middle-aged and have even lower old-age consumption. In all calibrations we consider in the following section, QH agents consume less when middle-aged than RE agents.

[5]See Appendix B for an overview of the optimization problem solved by QH agents.

Table 2: Baseline 1 Equilibrium Differences for RE and QHD Agents

RE	vs	QHD	Analytical Difference (RE - QHD)
c_y^{R*}	$<$	c_y^{Q*}	$\frac{1}{1+\beta+\beta^2+\gamma}\left(x+\frac{(1-\tau)W_u(1+\phi)}{1+r}\right)-\frac{1}{1+\delta\beta+\delta\beta^2+\gamma}\left(x+\frac{(1-\tau)W_u(1+\phi)}{1+r}\right)$
s^{R*}	$>$	s^{Q*}	$\frac{\gamma(1-\delta)(\beta+\beta^2)[(1-\tau)W_u(1+\phi)+(1+r)x]}{(1-\tau)W_u[\phi-(1+r)](1+\beta+\beta^2+\gamma)(1+\delta\beta+\delta\beta^2+\gamma)}$
c_m^{R*}	$>$	c_m^{Q*}	$\frac{(1-\delta)\beta[(1-\tau)W_u(1+\phi)+(1+r)x][(\gamma+1)-\delta(\beta^3+\beta^2)]}{(\beta\delta+1)(\delta\beta^2+\delta\beta+\gamma+1)(\beta^2+\beta+\gamma+1)}$
c_o^{R*}	$>$	c_o^{Q*}	$\frac{\beta^2(1-\delta)(1+r)[(1-\tau)W_u(1+\phi)+x(1+r)](\delta+\gamma+2\beta\delta+\delta\gamma+\beta^2\delta+\beta\delta\gamma+1)}{(\beta\delta+1)(\delta\beta^2+\delta\beta+\gamma+1)(\beta^2+\beta+\gamma+1)}$

Table 3 contains the analytical distinctions between expected (E) and actual (A) equilibrium consumption for quasi-hyperbolic discounting optimizers. As discussed above, agent re-optimization leads to an inequality between the planned consumption path and the realized equilibrium consumption path for optimizing QH agents . For all specifications of our parameters in the range outlined above, QH agents expect to consume more when old and less when middle-aged than they actually consume in equilibrium. This distinction creates a wedge between expected lifetime utility when young and realized lifetime utility. This wedge

Table 3: Baseline 1 Expected vs Realized Consumption for QHD Agents

QHD(E)	vs	QHD(A)	Analytical Difference)
$c_m^{Q'}$	$<$	c_m^{Q*}	$\frac{(\delta-1)\beta^2\delta((1-\tau)W_u(1+\phi)+(1+r)x)}{(\beta\delta+1)(\delta\beta^2+\delta\beta+\gamma+1)}$
$c_o^{Q'}$	$>$	c_o^{Q*}	$\frac{(1-\delta)(1+r)\beta^2\delta((1-\tau)W_u(1+\phi)+(1+r)x)}{(\beta\delta+1)(\delta\beta^2+\delta\beta+\gamma+1)}$

will be discussed further in the following section when we consider a slightly more complex model analytically.

We conclude this section with a brief analysis of the relationship between equilibrium consumption and schooling for rational exponential discounters with a high discount rate (RH), quasi-hyperbolic discounters, and rational exponential discounters that discount future utility at the rate β. An RH agent is an individual that discounts utility from consumption i periods into the future at the rate $(\beta\delta)^i$. Recall, a QH agent and an RE agent discount consumption utility i periods into the future at the rate $\delta\beta^i$ and β^i, respectively.

Populating our model with RH agents and optimizing leads to the following equilibrium orderings for schooling and consumption in each stage of life:

$$\begin{aligned} c_y^{RH^*} &> c_y^{Q^*} > c_y^{R^*} \\ s^{RH^*} &< s^{Q^*} < s^{R^*} \\ c_m^{RH^*} &< c_m^{Q^*} < c_m^{R^*} \\ c_o^{RH^*} &< c_o^{Q^*} < c_o^{R^*} \end{aligned}$$[6]

This analysis leads us to conclude that populating our model with high discounting rational agents and typical RE agents (as outlined above) acts as a bound on the equilibrium behavior of QH agents. However, the desire to re-optimize when middle-aged remains a unique feature associated with QH agents. Thus, a highly impatient rational agent with a high exponential discount factor will never look back with regret on their previous decisions, even though they consume a lot when young and have limited resources remaining when old. However, a QH agent will look back with regret, creating a break in one's realized utility relative to expected utility. It is precisely this gap that allows policy makers to design education incentives that improve the backward looking lifetime welfare of QH agents, outlined in section 4.

In the following section, we proceed by adding pay-as-you-go social security (PAYG) and an education incentive pay to our baseline model in an attempt to reduce the old age consumption gap that arises between QH and RE agents.

4. Policy Experiments: Social Security and Education Incentives

The set-up outlined in the previous section remains the same, but instead of assuming that all tax revenue is discarded, we assume that the government redistributes tax revenue via PAYG social security. The government balances its budget each period by distributing all

[6] When $\beta\delta \to 1$, $c_o^{Q^*} < c_o^{RH^*} < c_o^{R^*}$. However, discount rates of this magnitude are not empirically feasible, particularly for 3 period model in which a single period represents multiple years.

taxes levied on young and middle-aged workers to old agents via a lump sum social security transfer.

4.1 Social Security: RE Solution

A rational agent is faced with the same optimization problem outlined in the previous section. However, the old age budget constraint now includes social security transfers b:

$$c_{t+2}^t = (1+r)a_{t+1}^t + b_{t+2}^t \tag{14}$$

The government collects labor income taxes from both young and middle aged workers and distributes tax revenue to old agents in the following way: $b_{t+2}^t = \tau W_u(1-s_{t+2}^{t+2}) + \tau W_u(1+\phi s_{t+1}^{t+1})$ is the transfer received in period $t+2$ by generation t agents from generation $t+1$ agents (middle aged when generation t agents are old) and generation $t+2$ agents (young agents when generation t agents are old). Optimization leads to the following consumption and education profile:

$$\begin{aligned}
c_t^t &= \frac{1}{1+\beta+\beta^2+\gamma}\left[x + \frac{(1-\tau)W_u(1+\phi)}{1+r} + \frac{b_{t+2}}{(1+r)^2}\right] && (15)\\
c_{t+1}^t &= \beta(1+r)c_t^t && (16)\\
c_{t+2}^t &= \beta^2(1+r)^2 c_t^t && (17)\\
s_t^t &= 1 - \frac{\gamma(1+r)}{(1-\tau)W_u[\phi-(1+r)]}c_t^t && (18)
\end{aligned}$$

Unlike the baseline model outlined in the previous section, our equilibrium is not yet pinned down due to the inclusion of s_{t+1}^{t+1} and s_{t+2}^{t+2} in b_{t+2}^t. However, as all agents behave symmetrically in equilibrium, we know $s_{t+2}^{t+2} = s_{t+1}^{t+1} = s_t^t = s^{R^*}$. Thus, we replace the social security transfer received by old agents with its equilibrium value: $b^{R^*} = \tau W_u(1-s^{R^*}) +$

$\tau W_u(1+\phi s^{R^*})$. This leads to the following solution for equilibrium schooling:

$$s^{R^*} = \left(1+\xi\left((\phi-1)\frac{\tau W_u}{(1+r)^2}\right)\right)^{-1} \times \left(1-\xi\left(x+\frac{(1-\tau)W_u(1+\phi)}{1+r}+\frac{2\tau W_u}{(1+r)^2}\right)\right) \quad (19)$$

where $\xi = \frac{\gamma(1+r)}{(1-\tau)W_u[\phi-(1+r)]}\left(\frac{1}{1+\beta+\beta^2+\gamma}\right)$. Our equilibrium is given by s^{R^*} and:

$$b^{R^*} = \tau W_u(1-s^{R^*}) + \tau W_u(1+\phi s^{R^*}) \quad (20)$$

$$c_y^{R^*} = \frac{1}{1+\beta+\beta^2+\gamma}\left(x+\frac{(1-\tau)W_u(1+\phi)}{1+r}+\frac{b^{R^*}}{(1+r)^2}\right) \quad (21)$$

$$c_m^{R^*} = \beta(1+r)c_y^{R^*} \quad (22)$$

$$c_o^{R^*} = \beta^2(1+r)^2 c_y^{R^*} \quad (23)$$

Consumption in every period is an increasing function of the social security transfer b_{t+2}^t, however schooling is a decreasing function of the social security transfer b_{t+2}^t. Thus, in equilibrium social security plays a dual role in the determination of consumption. By raising lifetime income for a given schooling decision relative to the model in section 3a in which tax revenue is discarded, at first glance social security appears to increase equilibrium consumption. However, by disincentivizing investment in schooling, middle aged income is lower when taxes are levied and used to fund social security. We will discuss the net effect of social security on consumption and lifetime utility in a calibration exercise following our delineation of the QH solution below.

4.2. Social Security: QH Solution

We now consider the same optimization problem for a QH household. All distinctions arising between QH and RE households in the first period are identical to those outlined in

Section 3. The expected consumption profile for a QH agent is:

$$
\begin{aligned}
\tilde{c}_t^t &= \frac{1}{1+\delta\beta+\delta\beta^2+\gamma}\left[x+\frac{(1-\tau)W_u(1+\phi)}{1+r}+\frac{\tilde{b}_{t+2}^t}{(1+r)^2}\right] && (24)\\
\tilde{c}_{t+1}^{t'} &= \delta\beta(1+r)\tilde{c}_t^t && (25)\\
\tilde{c}_{t+2}^{t'} &= \delta\beta^2(1+r)^2\tilde{c}_t^t && (26)\\
\tilde{s}^t &= 1-\frac{\gamma(1+r)}{1-\tau W_u[\phi-(1+r)]}\tilde{c}_t^t && (27)
\end{aligned}
$$

We solve for s^{Q^*} by replacing $\tilde{b}_{t+2}^t$ with $b^{Q^*}=\tau W_u(1-s^{Q^*})+\tau W_u(1+\phi s^{Q^*})$ in equation (24) and substituting (24) into (27):

$$
s^{Q^*}=\left(1+\tilde{\xi}\left((\phi-1)\frac{\tau W_u}{(1+r)^2}\right)\right)^{-1}\times\left(1-\tilde{\xi}\left(x+\frac{(1-\tau)W_u(1+\phi)}{1+r}+\frac{2\tau W_u}{(1+r)^2}\right)\right) \quad (28)
$$

Where $\tilde{\xi}=\frac{\gamma(1+r)}{(1-\tau)W_u[\phi-(1+r)]}\left(\frac{1}{1+\delta\beta+\delta\beta^2+\gamma}\right)$. This leads to the following equilibrium social security transfer and expected consumption profile for QH optimizers:

$$
\begin{aligned}
b^{Q^*} &= \tau W_u(1-s^{Q^*})+\tau W_u(1+\phi s^{Q^*}) && (29)\\
c_y^{Q^*} &= \frac{1}{1+\delta\beta+\delta\beta^2+\gamma}\left[x+\frac{(1-\tau)W_u(1+\phi)}{1+r}+\frac{\tilde{b}^{Q^*}}{(1+r)^2}\right] && (30)\\
c_m^{Q'} &= \delta\beta(1+r)c_y^{Q^*} && (31)\\
c_o^{Q'} &= \delta\beta^2(1+r)^2c_y^{Q^*} && (32)
\end{aligned}
$$

As in the previous section, our QH agents will not follow through with their expected consumption profile when middle-aged. Thus, we must account for the re-optimization of QH agents in our realized equilibrium consumption profile. The actual equilibrium consumption profile for middle-aged and old agents is given by:

$$
\begin{aligned}
c_m^{Q^*} &= \frac{1}{1+\beta\delta}\left[(1+r)a_y^{Q^*}+(1-\tau)W_u(1+\phi s^{Q^*})+\frac{b^{Q^*}}{1+r}\right] && (33)\\
c_o^{Q^*} &= \beta\delta(1+r)c_m^{Q^*} && (34)
\end{aligned}
$$

An analytical approach aimed at comparing this collection and the equilibrium consumption, education, and government transfers associated with RE optimization is no longer simple enough to illuminate the distinctions that arise between our two types of agents. Therefore, we proceed by calibrating our model in order to characterize the equilibrium distinctions that arise between RE and QH optimizers.

4.3. Social Security: RE vs QH Outcomes

The return to education parameter, $\phi = 2.6$, was calculated by imposing our linear human capital production function, $h_{t+1} = (1 + \phi s_t)$, and estimating the implied return to schooling using 2015 BLS data on median weekly earnings for high school dropouts ($s = 0$), high school graduates ($s = 0.2$), associate degree holders ($s = 0.4$), bachelor degree holders ($s = 0.6$), masters degree holders ($s = 0.8$) and professional degree holders ($s = 1$). As stated previously, we abstract from the kinked nature of this human capital production function and instead chose to average over the implied values of ϕ. Unskilled wage $W_u = \$235,000$ as this represent the income a full time low-skill worker (no high school diploma) would earn over 10 years according to the same 2015 BLS data used to calculate ϕ. We chose $\beta = 0.66$ which corresponds to an annual $\beta = 0.96$ raised to the 10^{th}, $r = 0.515$ so that $\beta(1 + r) = 1$, and $\gamma = 0.5$. We set our agents' inheritance $x = 0$ in our baseline specification.

Table 4 outlines the differences in equilibrium behavior for RE and QH societies. Differences in expected utility for QH optimizers are also included at the bottom of Table 4. Our preferred value of δ, the present bias parameter, is 0.7 corresponding to the findings of Laibson et al. (2007). However, we choose to use values of $\delta = 0.8$ and $\delta = 0.6$ in order to explicate the impact of a high degree of present bias as well as a low degree of present bias that are conveniently centered around our preferred parameterization. We will implement $\delta = 0.7$ in our analysis in Section 4.6 comparing social security to an education incentive program.

Table 4: III-B Calibrated S.S. Differences for RE and QH agents

δ	.8						.6					
τ	0.0		0.1		0.2		0.0		0.1		0.2	
		%Δ		%Δ		%Δ		%Δ		%Δ		%Δ
s^{R^*}	0.36	16.3%	0.33	18.5%	0.29	21.6%	0.36	36.0%	0.33	40.6%	0.29	47.4%
s^{Q^*}	0.30		0.27		0.23		0.23		0.19		0.15	
$c_y^{R^*}$	2.15	9.2%	2.04	9.0%	1.91	8.8%	2.15	20.3%	2.04	19.8%	1.91	19.2%
$c_y^{Q^*}$	2.35		2.22		2.08		2.59		2.44		2.28	
$c_m^{R^*}$	2.15	5.1%	2.04	5.2%	1.91	5.5%	2.15	14.1%	2.04	14.5%	1.91	14.9%
$c_m^{Q^*}$	2.04		01.93		1.81		1.85		1.74		1.63	
$c_o^{R^*}$	2.15	-24.0%	2.04	-24.2%	1.91	-24.4%	2.15	-48.5%	2.04	-48.7%	1.91	-50.0%
$c_o^{Q^*}$	1.63		1.54		1.45		1.11		1.04		0.98	
b^{R^*}	0.00	N/A	0.59	3.8%	1.16	4.0%	0.00	N/A	0.59	8.4%	1.16	8.9%
b^{Q^*}	0.00		0.57		1.11		0.00		0.54		1.05	
$E(U_y^{Q'})$	13.305	-.0483%	13.242	-.0485%	13.173	-.0488%	13.096	-0.250%	13.031	-0.251%	12.959	-0.253%
$E(U_y^{Q^*})$	13.298		13.236		13.167		13.063		12.998		12.926	

All values of consumption, c, and government transfers, b, stated above are in hundreds of thousands of units.

Note: In the table above, for all equilibrium values the %Δ is calculated as $\frac{x^{Q^*} - x^{R^*}}{x^{R^*}}$.

Our measure of expected utility, the variables $E(U_y^{Q'})$ and $E(U_y^{Q^*})$, represent the utility associated with the expected consumption profile solved for by a young QH agent and the utility associated with the actual consumption profile that a QH agent will choose after re-optimizing when middle aged, respectively. These measures of utility are both calculated from the perspective of a young agent, so as to provide a measure of the utility loss associated with re-optimization performed by a QH agent.

We focus on changes in consumption and percent differences between consumption profiles for RE and QH agents. When $\tau = 0$, the results in table 4 are merely a calibrated version of the equilibrium in Section 3. For each tax rate τ and QH discount rate δ considered, the analytical results outlined in Table 2 hold even with the inclusion of PAYG social security. We vary over several tax rates (and thus transfer amounts) and find that the gap between optimal rational and optimal QH behavior is decreasing in the discount factor δ. That is, the more present biased a society is, the larger the gap between it's equilibrium behavior and equilibrium behavior in a rational society[7].

For a given δ, increasing the role of social security (via increasing τ) leads to lower education for both RE and QH agents and a larger education gap between RE and QH societies. This comes from the inherent overweighting of current utility relative to future utility that distinguishes QH agents from RE agents. When RE and QH agents receive an increase to their old age income via a social security transfer, they immediately disinvest in education while young and reduce their planned savings when middle aged. However, QH agents do so at a much higher rate than RE agents, which leads to lower relative human capital and lifetime earnings.

Increasing social security decreases consumption in each period of life for RE and QH agents. Utility is also lower for all agents when taxes increase. This finding is in line with the majority of the literature regarding the welfare reducing effects of social security in overlapping generations models with human capital. When population growth is zero and

[7] Although table 4 only highlights the use of two parameterizations of δ, numerical analysis corroborates these findings (the higher δ, the smaller the gap between RE and QH optimization) $\forall\ \delta \in (0, 1)$

returns from other investments (specifically education) are positive, PAYG social security is a less effective means of saving than alternative investment options. Further, the utility gap between expected equilibrium consumption and actual equilibrium consumption of a young agent is increasing in the tax rate. This is outlined by the variables $E(U_y^{Q'})$ and $E(U_y^{Q^*})$ reported in the bottom two rows of Table 4. These variables are calculated by evaluating utility from consumption in the final two periods of life (from the perspective of a young agent) using actual equilibrium consumption ($c_m^{Q^*}$ and $c_o^{Q^*}$) to calculate $E(U_y^{Q^*})$ and planned equilibrium consumption ($c_m^{Q'}$ and $c_o^{Q'}$) to calculate $E(U_y^{Q'})$. Both realized and planned consumption utility are decreasing in the tax rate for a given value of δ when social security is the only government outlay. From the perspective of a policy maker who is unsure whether agents in her society are rational or present biased, social security proves to be an unequivocally poor policy intervention. It is both welfare reducing for RE and QH agents, it increases the old age consumption gap between these two theoretical societies, and it increases the utility gap between expected and realized consumption for a QH agent.

Thus far we have outlined distinctions that arise between RE and QH societies without discussing the likelihood that members of a society belong to one group or the other. The lifetime consumption profile of QH agents provides evidence that QH optimization may help to explain some features of observed consumption behavior that RE optimization cannot explain. The drop in old age consumption observed in time-series of U.S. data cannot be remedied with a model of RE optimization without the inclusion of several constraints, but it arises naturally in our simple 3-period model when optimizers are present biased. Although this is not resounding evidence that societies behave in a systematically present biased way, it is a feature of QH optimization that provides merit to the analysis of QH societies.

4.4. Social Security and Education Incentives: RE Solution

In the following section, we augment the previous model by considering a second channel for the distribution of tax revenue. As social security unequivocally lowered household human capital and utility relative to a baseline in which the government does not levy taxes, we consider an alternative avenue for government spending. Agents are faced with the same optimization problem outlined in Section 4.1, but we replace the budget constraint of young agents with:

$$c_t^t + a_t^t = x + i_t^t s_t^t + (1-\tau)W_u(1-s_t^t) \tag{35}$$

where i_t^t is the incentive pay provided to students by the government. As before, the government taxes agents' labor income at the rate τ. Total tax revenue collected in period t remains the same. However, instead of directly transferring said revenue to old agents, the government splits its receipts between funding social security and education incentive pay. To maintain a balanced budget in every period, the government divides tax revenue in the following way:

$$i_t^t = \alpha[\tau W_u(1-s_t^t) + \tau W_u(1+\phi s_{t-1}^{t-1})] \tag{36}$$

$$b_t^{t-2} = (1-\alpha)[\tau W_u(1-s_t^t) + \tau W_u(1+\phi s_{t-1}^{t-1})] + \alpha(1-s_t^t)i_t^t \tag{37}$$

where $\alpha \in [0,1]$. The government collects sufficient taxes so that if our representative agent chooses $s_t^t = 1$, the government can afford to pay her the education incentive pay for her entire schooling investment. If our agent chooses $s_t^t < 1$, then the excess revenue the government collects, $(1-s_t^t)i_t^t$, is added to the social security transferred to old agents. It is important to note that although we have referred to the education of the representative agent of generation t as s_t^t, this agent is actually an atomistic member of their generation. Thus, a young agent does not internalize the impact of their schooling decision on the gross education incentive pay, i_t.

Optimization leads to the following equilibrium consumption and education profiles for an agent born in generation t:

$$c_t^t = \frac{1}{1+\beta+\beta^2+\gamma}\left[x+i_t+\frac{(1-\tau)W_u(1+\phi)}{1+r}+\frac{b_{t+2}}{(1+r)^2}\right] \quad (38)$$

$$c_{t+1}^t = \beta(1+r)c_t^i \quad (39)$$

$$c_{t+2}^t = \beta^2(1+r)^2c_t^i \quad (40)$$

$$s_t^t = 1-\frac{\gamma(1+r)}{(1-\tau)W_u[\phi-(1+r)]+(1+r)i_t}c_t^t \quad (41)$$

As in Section 4.1 we must now account for the inclusion of s in both i_t and b_{t+2}. In equilibrium, i^{R^*} will be given by $\alpha[\tau W_u(1-s^{R^*})+\tau W_u(1+\phi s^{R^*})]$ and b^{R^*} will be given by $(1-\alpha)[\tau W_u(1-s^{R^*})+\tau W_u(1+\phi s^{R^*})]+(1-s^{R^*})i^{R^*}$. Thus, we do not yet have our equilibrium profile completely solved as s^{R^*} has not been pinned down. We substitute in for c_t^t in equation (38) and sub in i^{R^*} and b^{R^*} according to the above definition. This leads to the following equilibrium consumption, schooling, and government spending profile:

$$i^{R^*} = \alpha[\tau W_u(1-s^{R^*})+\tau W_u(1+\phi s^{R^*})] \quad (42)$$

$$b^{R^*} = (1-\alpha)[\tau W_u(1-s^{R^*})+\tau W_u(1+\phi s^{R^*})]+(1-s^{R^*})i^{R^*} \quad (43)$$

$$c_y^{R^*} = \frac{1}{1+\beta+\beta^2+\gamma}\left[x+i^{R^*}+\frac{(1-\tau)W_u(1+\phi)}{1+r}+\frac{b^{R^*}}{(1+r)^2}\right] \quad (44)$$

$$c_m^{R^*} = \beta(1+r)c_y^{R^*} \quad (45)$$

$$c_o^{R^*} = \beta^2(1+r)^2c_y^{R^*} \quad (46)$$

$$s^{R^*} = 1-\Xi\left(x+i^{R^*}+\frac{(1-\tau)W_u(1+\phi)}{1+r}+\frac{b^{R^*}}{(1+r)^2}\right) \quad (47)$$

where $\Xi = \dfrac{\gamma(1+r)}{((1-\tau)W_u[\phi-(1+r)]+(1+r)i^{R^*})(1+\beta+\beta^2+\gamma)}$.

4.5. Social Security and Education Incentives: QH Solution

Populating our model with QH agents leads to the same issue outlined in above in regards to analytically deriving equilibrium schooling.

$$s^{Q^*} = 1 - \tilde{\tilde{\Xi}}\left(x + i^{Q^*} + \frac{(1-\tau)W_u(1+\phi)}{1+r} + \frac{b^{Q^*}}{(1+r)^2}\right) \tag{48}$$

Where $\tilde{\tilde{\Xi}} = \dfrac{\gamma(1+r)}{((1-\tau)W_u[\phi-(1+r)]+(1+r)i^{Q^*})(1+\delta\beta+\delta\beta^2+\gamma)}$

This leaves us with the following equilibrium government transfers and consumption profile for a QH agent:

$$i^{Q^*} = \alpha[\tau W_u(1-s^{Q^*}) + \tau W_u(1+\phi s^{Q^*})] \tag{49}$$

$$b^{Q^*} = (1-\alpha)[\tau W_u(1-s^{Q^*}) + \tau W_u(1+\phi s^{R^*})] + (1-s^{Q^*})i^{Q^*} \tag{50}$$

$$c_y^{Q^*} = \frac{1}{1+\delta\beta+\delta\beta^2+\gamma}\left[x + i^{Q^*} + \frac{(1-\tau)W_u(1+\phi)}{1+r} + \frac{b^{Q^*}}{(1+r)^2}\right] \tag{51}$$

$$c_m^{Q^*} = \frac{1}{1+\beta\delta}\left[(1+r)a_y^{Q^*} + (1-\tau)W_u(1+\phi s^{Q^*}) + \frac{b^{Q^*}}{1+r}\right] \tag{52}$$

$$c_o^{Q^*} = \beta\delta(1+r)c_m^{Q^*} \tag{53}$$

where $a_y^{Q^*} = x + i^{Q^*}s^{Q^*} + (1-\tau)W_u(1-s^{Q^*}) - c_y^{Q^*}$.

4.6. Social Security and Education Incentives: RE vs QH Outcomes

Table 5 outlines the equilibrium consumption and education profiles for RE and QH agents. Rather than varying the degree of present biasedness (as in Table 4), we instead focus on the tax rate and the allocation of taxes between funding education incentives and PAYG social security. We chose a value of $\delta = 0.7$ for our QH agents as this is our preferred value of δ. When $\alpha = 0$, the calibrations in Table 5 represent the equilibrium consumption and schooling profile of RE and QH agents who only receive social security payments. Thus, columns 1,2 and 8 of table 5 are simply calibrated equilibrium profiles for model 2 when $\delta = 0.7$ and are readily comparable to the calibrations outlined in table 4.

Table 5 clearly shows if the government is going to levy taxes, both consumption and schooling are higher for RE and QH agents in every period of life when $\alpha = 1$ relative to $\alpha < 1$. Unlike our exposition of of PAYG social security in Section 4.3, taxes no longer

Table 5: Baseline 3 Calibrated S.S. Differences for RE and QH agents

τ	0.0		0.1						0.2					
α	n/a		0.0		0.5		1.0		0.0		0.5		1.0	
		%Δ		%Δ		%Δ		%Δ		%Δ		%Δ		
s^{R^*}	0.36	25.7%	0.33	29.0%	0.41	21.5%	0.48	17.0%	0.29	33.0%	0.46	18.2%	0.57	12.5%
s^{Q^*}	0.27		0.23		0.32		0.40		0.19		0.38		0.50	
$c_y^{R^*}$	2.15	14.5%	2.04	14.2%	2.14	14.1%	2.24	14.2%	1.91	13.8%	2.13	13.9%	2.34	14.1%
$c_y^{Q^*}$	2.46		2.34		2.44		2.56		2.18		2.43		2.70	
$c_m^{R^*}$	2.15	9.0%	2.04	9.3%	2.14	9.3%	2.24	9.2%	1.91	9.6%	2.13	9.5%	2.34	9.3%
$c_m^{Q^*}$	1.96		1.85		1.94		2.04		1.73		1.93		2.13	
$c_o^{R^*}$	2.15	-36.3%	2.04	-36.5%	2.14	-36.5%	2.24	-36.5%	1.91	-36.7%	2.13	-36.6%	2.35	-36.5%
$c_o^{Q^*}$	1.37		1.29		1.36		1.42		1.21		01.35		1.49	
b^{R^*}	0	N/A	0.59	6.0	0.50	0.0%	0.34	10.3%	1.16	6.4%	0.99	0.3%	0.59	11.8%
b^{Q^*}	0		0.56		0.50		0.37		1.08		0.99		0.66	
i^{R^*}	0	N/A	0.00	N/A	0.31	-5.3%	0.65	-4.7%	0.00	N/A	0.64	-4.9%	1.37	-4.9%
i^{Q^*}	0		0.00		0.29		0.62		0.00		0.61		1.31	
$E(U_y^{Q'})$	13.210	123%	13.147	- 123%	13.201	- 123%	13.253	- 122%	13.076	- 124%	13.196	- 123%	13.301	- 122%
$E(U_y^{Q^*})$	13.194		13.130		13.185		13.237		13.060		13.180		13.285	

All values of consumption, c, and government transfers, b and i, stated above are in hundreds of thousands of units.

Note: In the table above, for all equilibrium values the %Δ is calculated as $\dfrac{x^{Q^*} - x^{R^*}}{x^{R^*}}$.

unequivocally lower consumption and education for RE or QH agents. Rather, higher taxes in conjunction with education incentive pay lead to higher equilibrium consumption and schooling relative to no taxation. When $\alpha = 1$, equilibrium schooling is over a year higher for both RE and QH agents for $\tau = 0.1$ and over two years higher for RE and QH agents when $\tau = 0.2$ relative to a regime with no taxation. Further, QH agents respond more drastically to higher education incentive pay than RE agents. This can be seen by comparing the schooling gap between RE and QH agents for a respective tax rate when $\alpha = 0$ and when $\alpha = 1$. Although we ignore endogenous growth and other positive spillovers from education in this paper, Table 5 provides preliminary evidence that dedicating revenue from labor income taxation to education incentive pay leads to drastically higher schooling and consumption than an equivalent income tax rate with all proceeds being directed to PAYG social security.

As in Table 4, we are capable of comparing utility from expected consumption when middle aged and old to utility from actual consumption when middle aged and old from the perspective of a young QH agent. When taxes are used for PAYG social security, a higher tax rate corresponds to lower utility from both expected and realized middle age and old age consumption and a higher utility gap between expected and realized consumption utility (see Table 4 rows 13 and 14). When the government splits tax revenue between PAYG social security and education incentive pay, the gap between expected and realized consumption utility is decreasing in α. That is, the higher the percentage of tax revenue dedicated to education incentive pay, the smaller the gap between utility associated with expected middle aged and old consumption and utility associated with actual middle aged and old consumption for a QH agent. Further, when $\alpha = 1$, utility from expected consumption and realized consumption is higher than when $\alpha < 1$ and $\tau = 0$. Thus, not only is government funded education inventive pay welfare improving relative to social security alone for a given tax rate, it is also welfare improving *relative to no taxation!*

Numerical analysis confirms that equilibrium utility is higher under linear education pay with $\alpha = 1$ for both QH and RE agents when viewed from the perspective of being young, middle aged, or old relative to no taxation. Thus, for a policy maker in either a QH or RE society, our linear education pay is a simple policy measure that can be enacted in order to increase lifetime consumption, education, and utility of optimizing agents.[8]

IV. Conclusion and Future Work

We build a unique model of human capital investment and analyze the optimizing behavior of both traditional exponential discounters and quasi-hyperbolic discounters. We find that agents in a quasi-hyperbolic society accumulate less human capital, consumes more when young and save less for retirement than agents in a rational, exponentially discounting society. The steep drop in retirement consumption generated by our simple three period OLG model with no idiosyncratic risk and perfect capital markets in the presence of QH agents provides further evidence that present bias behavior matches certain empirical regularities [9]

We analyze several different arrangements for the disbursement of tax revenue. We find that both RE and QH societies are made worse off when taxes are levied and tax revenue is dedicated to funding PAYG social security relative to a regime in which no taxes are levied. Agents have lower utility, lower consumption in each period, and obtain less schooling relative to a tax free regime. Further, the gap between expected future utility from consumption and realized utility from consumption is increasing for QH agents in both the tax rate (τ) and the degree to which agents are present biased (δ). However, taxes can be welfare improving if tax revenue is split between funding education incentive pay and social security. In section 4 we show that agents in both RE and QH societies obtain more schooling and consume more in each period of life when taxes are dedicated to education incentive pay. Further, education incentive pay reduces the welfare gap between expected and realized utility from consumption for QH agents in our preferred specification.

Future work will be dedicated to understanding the implications of agent heterogeneity in the model outlined above. We seek to understand the role of idiosyncratic income shocks

[8]Note: we have assumed prices are exogenous. Changing this assumption could change the impact of equilibrium government intervention on both QH and RE agent's utility. We leave this analysis for future work.

[9]See Angeletos et al. 2001 for further exposition on this topic.

that correlate with agent human capital in a world in which agents have different initial endowments. After outlining the role of present bias in generating wealth inequality, we will analyze the efficacy of the government spending regimes discussed above for reducing wealth inequality in both QH and RE societies.

Appendix Solution to the household optimization problem:

We assume $a^t_{t-i} \in \mathbb{R} \ \forall \ i \in \mathbb{Z}$. Thus, we can combine the three budget constraints given by (1)-(3) in the text into a single budget constraint via our agent's asset holdings:

$$c^t_t + \frac{c^t_{t+1}}{1+r} + \frac{c^t_{t+2}}{(1+r)^2} = x + (1-\tau)W_u(1-s^t_t) + \frac{(1-\tau)W_u(1+\phi s^t_t)}{1+r} \tag{54}$$

We are now able to express the agent's problem as a Lagrangian.

$$\mathscr{L} = \ln(c^t_t) + \gamma \ln(1-s^t_t) + \beta \ln(c^t_{t+1}) + \beta^2 \ln(c^t_{t+1})$$

$$+ \lambda^t_t \left(x + (1-\tau)W_u(1-s^t_t) + \frac{(1-\tau)W_u(1+\phi s^t_t)}{1+r} - c^t_t - \frac{c^t_{t+1}}{1+r} - \frac{c^t_{t+2}}{(1+r)^2} \right)$$

Optimization yields the following first order conditions:

$$\mathscr{L}_1 = 0 \implies \frac{1}{c^t_t} = \lambda^t_t \tag{55}$$

$$\mathscr{L}_2 = 0 \implies \frac{\beta}{c^t_{t+1}} = \frac{\lambda^t_t}{1+r} \tag{56}$$

$$\mathscr{L}_3 = 0 \implies \frac{\beta^2}{c^t_{t+2}} = \frac{\lambda^t_t}{(1+r)^2} \tag{57}$$

$$\mathscr{L}_4 = 0 \implies -\frac{\gamma}{1-s^t_t} - \lambda^t_t(1-\tau)W_u + \lambda^t_t \frac{(1-\tau)W_u\phi}{1+r} = 0 \tag{58}$$

$$\mathscr{L}_5 = 0 \implies \text{Budget Constraint (equation (4))} \tag{59}$$

Re-optimizing in (t+1) for QH agents

$$Max \ \tilde{U}_{t+1} = \ln(\tilde{c}^t_{t+1}) + \delta\beta \ln(\tilde{c}^t_{t+2})$$

s.t.

$$\tilde{c}^t_{t+1} + \tilde{a}^t_{t+1} = (1-\tau)W_u(1+\phi s^{Q^*}) + \tilde{a}^t_y(1+r) \tag{60}$$

$$\tilde{c}^t_{t+2} = (1+r)\tilde{a}^t_{t+1} \tag{61}$$

(60) can be thought of as the constrained analogue to (2) in which a middle-aged agent re-optimizes taking the choice of their younger self as given.

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