Behavioral Biases in General Equilibrium*

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Abstract

This paper studies present bias in a life-cycle model with heterogeneous preferences. Previous research outlining the role of behavioral biases in the macroeconomy largely falls into one of three categories: (1) partial equilibrium, (2) infinite horizon, or (3) homogeneous preferences. We introduce three novel findings to this literature. First, partial equilibrium models dramatically overstate the impact of present bias on economy-wide and household outcomes. Second, the life-cycle is an essential element for understanding the impact of present bias on agent decision making. Third, when considering behavioral biases in general equilibrium, the proportion of agents with present-biased preferences is a crucial determinant of the economy-wide wealth distribution.

JEL Classification: D15, E21, E71, G51
Keywords: Present Bias, Wealth Inequality, Life-cycle Model, Preference Heterogeneity

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1 Introduction

Present bias[1] refers to a preference anomaly wherein individuals discount tradeoffs between all future periods and the current period at a higher rate than they discount tradeoffs between any two periods in the future. First outlined by Phelps and Pollak (1968) and Strotz (1956) and later reintroduced to the economics literature by Laibson (1997), present bias offers a convenient theoretical framework for explaining discrepancies between intentions and actions of optimizing individuals. Even when individuals have perfect information regarding the future, present bias creates a time inconsistency in preferences that leads individuals to abandon any planned decision in favor of greater within period utility relative to their previous plan.

This paper examines the avenues through which this consistent re-optimization impacts aggregate wealth and the timing of wealth accumulation over the life-cycle. There is a robust empirical literature outlining the degree to which individuals are present-biased in both laboratory settings and over real world trade-offs.[2] This has led to a growing macroeconomic literature that considers behavioral biases, and present bias in particular, in life-cycle or neoclassical models.

This literature primarily focuses on the effects of present biased discounters in a partial equilibrium framework (Angeletos et al., 2001; Harris and Laibson, 2001; Laibson et al., 2020) or considers economies populated by homogeneous agents all with present-biased preferences (İmrohoroğlu et al., 2003; Krussell et al., 2002). A prominent exception is Maliar and Maliar (2006) which embeds both present-biased and exponential discounters into the same neoclassical growth economy. However, by modeling infinitely lived agents the crucial interaction of short planning horizons and present bias is not a feature of their analysis.[3] I extend the literature by embedding both exponential discounters and present-biased agents into general equilibrium life-cycle model economy.

Three key results stand out from my analysis. First, if present-biased agents are embedded in a partial equilibrium model, the impact of time inconsistency will be drastically overstated. Second, present bias increases wealth inequality as cohorts age and changes the timing of wealth accumulation in the model economy relative to a baseline model in which all agents behave rationally. Models that abstract from the richness of a life-cycle and instead place agents in an infinite horizon context will understate the role of present bias in the macroeconomy. Third, the inclusion of discount rate and present bias heterogeneity improves the fit of the model economy to U.S. data. Although a number of authors have shown heterogeneous exponential discount factors lead to increased wealth dispersion in a life-cycle model[4], to the best of my knowledge this paper marks the first attempt to characterize the contribution of heterogeneous present bias to aggregate economic outcomes in a life-cycle framework.

The remainder of the paper is structured as follows: Section 2 sets up the model. Section 3 provides an overview of how present bias impacts decision making. Section 4 outlines

[1]Also referred to as quasi-hyperbolic, quasi-geometric, or hyperbolic discounting in the literature.
[2]See Della Vigna, 2009; Meier and Sprenger, 2015; Paserman, 2008; Tanaka et al., 2010
[3]See Section 3 for further exposition regarding this mechanism.
[4]See Krussell and Smith, 1998; Hendricks, 2007.

our model calibration and section 5 outlines our experiments. Section 6 reports our results. Section 7 concludes and offers avenues for future study .

2 The Model

The modeling environment is a standard incomplete markets life-cycle model of the kind popularized by Huggett (1996). Insights from Hendricks (2007) and İmrohoroğlu et al. (2003) are utilized for modeling discount rate heterogeneity and present bias, respectively. The economy is comprised of a continuum of agents of unit mass, a perfectly competitive representative firm, and a government responsible for levying taxes and distributing social security.

An agent is born at model age 1, works for the first R periods of life, and dies with certainty after N>R periods. When an agent dies, they are replaced by a child of age 1 who inherits the after tax value of their parents' wealth and imperfectly inherits their parents' preferences and labor endowments. Agents inelastically supply $l = h(t)e$ units of labor to the market each period from birth to retirement, where $h(t)$ is a deterministic age-earnings profile and e is a labor endowment shock. Upon retirement, all agents receive an identical social security transfer, τ_R.

Agents share identical preferences with the exception of their discount factors. Upon drawing preference parameters, an agent has a state vector x given by $x = (k, e, t, j)$ where k is wealth, e is the current period's labor endowment shock, t is agent age, and j is a vector of discount factors, β and δ. The agent's optimization problem can be written as a dynamic program where the Bellman equation is given by:

$$V(x) = \max_{(c,\ k')} \ u(c) + \delta_j \beta_j s' \mathrm{E}\big[\tilde{V}(x'|x)\big] \tag{1}$$

subject to

$$c + k' \leq (1+r)k + wl + \tilde{\tau} + \tau_R, \tag{2}$$

$$k' \geq \underline{k},\ \ c \geq 0,\ \ k' > 0 \text{ if } t = N,\ \ V(x) = \tilde{V}(x) = 0 \text{ if } t = N+1. \tag{3}$$

where s is the age-dependent survival probability. δ_j and β_j are the present-biased and exponential discount factors of a type j agent[5]. The wage w and interest rate r are jointly determined by the profit maximization of perfectly competitive firms. All bequests are assumed to be unintentional.[6]

Present-biased agents ($\delta_j < 1$) are assumed to be naive, meaning they solve their dynamic programming problem under the belief that they will act in a time-consistent manner in the

[5] I have reversed the meaning of the β and δ parameters from Laibson's exposition of quasi-hyperbolic discounting so that the discount factor β retains its standard interpretation found throughout the macroeconomic literature.

[6] Previous research has shown that intentional bequests play a small role in life-cycle model economies in the absence of nonlinear bequest motives. See DeNardi (2004) and DeNardi and Yang (2014).

future.[7] This assumption can be represented by defining the continuation payoff $\tilde{V}(x)$ as:

$$\tilde{V}(x) = \max_{(c,\ k')}\ u(c) + \beta_j s' \mathrm{E}\big[\tilde{V}(x'|x)\big] \tag{4}$$

The continuation payoff of a present-biased agent, which informs an individuals' consumption-savings decisions over all future periods, is identical to the value function of an exponential discounter. The only distinction between an agent with $\delta_j < 1$ and an agent with $\delta_j = 1$ is the additional discounting of future utility made every period. It is exactly this additional discount factor that leads present-biased agents to behave inconsistently. For further detail regarding thd modeling environment, see Appendix A which outlines the steady state equilibrium of our model.

3 Present Bias: Inspecting the Mechanism

If individual's are present-biased then their discount factor is no longer constant. Rather, the discount factor becomes an endogenous variable that depends on an agent's current state, x. I denote the effective discount factor as $\beta_{x'}$ to highlight the new dependence of discounting on the state next period, x'. The effective discount factor is equal to[8]:

$$\beta_{x'} \equiv \beta_{x'}(k_{t+1}, e_{t+1}, t+1) = \beta\left[1 - \frac{1-\delta}{1+r}\frac{E_t[u'(c_{t+1})c_k(k_{t+1}, e_{t+1}, t+1)]}{E_t[u'(c_{t+1})]}\right] \tag{5}$$

where $c_k(k_{t+1}, e_{t+1}, t+1)$ is the derivative of the optimal consumption function with respect to k_{t+1}. When $\delta = 1$, $\beta_{x'} = \beta$ in each period. However, when $\delta < 1$, the effective discount factor is a function of the state in period $t+1$[9]. This generates a direct relationship between the effective discount factor and an agent's wealth, k. As shown in Maliar and Maliar (2006), if the consumption function is strictly concave, then the effective discount factor of present-biased agents will be strictly increasing in wealth. Thus, if two agents have the same degree of present bias but different levels of wealth, the richer of the two agents will behave more patiently than the poorer agent.

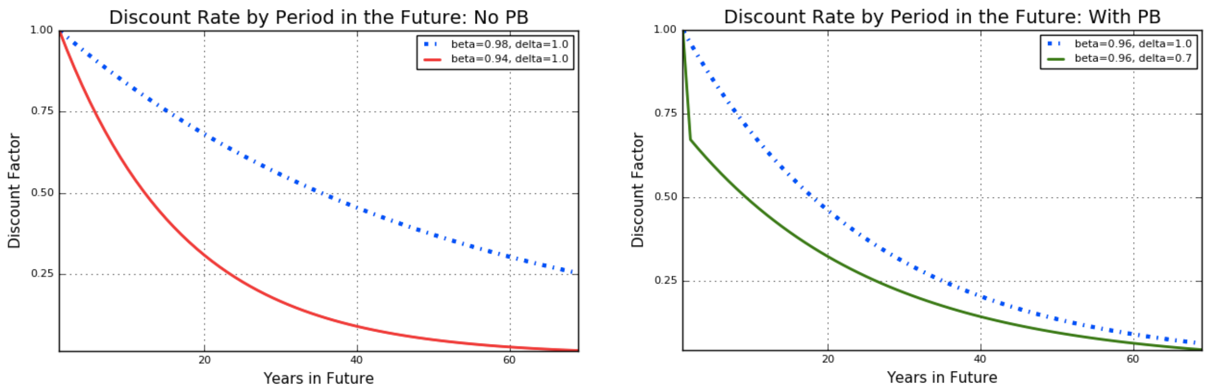
This result is fairly intuitive given diminishing marginal returns to consumption are a standard feature of individual utility functions. A present-biased agents ends each period with a consumption savings plan laid out for their future self only to wake up the next period and violate this plan. However, individuals who are very wealthy have less of an incentive to deviate from their planned consumption profile as their marginal utility gains are much smaller than those of a similarly biased individual with lower wealth. It is this exact relationship between effective discounting and wealth that leads to increased dispersion in savings between rich and poor present-biased households.

[7] Alternatively, one could model sophisticated agents who seek commitment devices to keep their future self from violating their planned consumption, savings profile. I assume agents are naive as there is no commitment device in my modeling environment and there is not a strong consensus in the behavioral literature regarding the true nature of agent sophistication. In fact, Laibson (2015) concludes "a demand for commitment is a special case rather than the general case".

[8] For further exposition of this derivation, see Appendix B.

[9] For a full exposition regarding the effective discount factor under quasi-hyperbolic discounting, see Harris and Laibson (2001).

A second avenue through which present bias impacts the accumulation of capital is the horizon over which savings decisions are made. Consider Figure 1 which shows the effective discount rate applied to utility n periods in the future. Panel (a) shows the discount rate for exponential discounters ($\delta = 1$) with $\beta = 0.98$ and 0.94, respectively. The impact of heterogeneity in the exponential discount factor leads to small discrepancies in discounting over a short planning horizon but large discrepancies in discounting over trade-offs further into the future. This heterogeneity leads to the creation of savers and spenders in the model, as agents all face the same interest rate. This intuition is confirmed in section 6, where I find the inclusion of heterogeneity in the exponential discount factor leads to a level shift in wealth inequality at each age in the model economy relative to a baseline model populated with homogeneous exponential discounters.



(a) homogeneous δ, heterogeneous β

(b) heterogeneous δ, homogeneous β

Figure 1: Discounting of Future Utility

Panel (b) shows the discount rate for agents with the same exponential discount factor ($\beta = 0.96$) and differing present bias $\delta = 1.0$ and 0.7, respectively. Unlike heterogeneity in the exponential discount factor, present bias creates the largest disagreements regarding time discounting over short planning horizons. It is this exact feature that leads to stark differences in wealth accumulation in old age between present biased and exponential discounters, making the life-cycle a crucial component of understanding present-biased behavior.

To recap, the impact of present bias on consumer decision making is decreasing in agent wealth. Thus, in an economy comprised of identical present-biased agents, there will still be a degree of effective preference heterogeneity if individuals have accumulated different amounts of wealth. Further, when individuals are present biased, the gap between their discounting of the future relative to an exponential discounter is largest when planning horizons are short. Thus, older households with little wealth will be most impacted by time-inconsistent preferences.

4 Model Calibration

In this section, we provide an overview of the model calibration for the parameters governing the behavior of households, firms, and the government.

4.1 Demographics

Households are born at the age of 22, retire and begin to receive social security at age 65, and live at most 69 periods (age 90). The probability of survival from one year to the next (P_s) is given by the mortality rates listed in the Social Security Life Tables [10].

Table 1: Model Parameters

Demographics	
$a_D = 69$	Maximum lifetime (Physical age of 90)
$a_R = 43$	Retirement age (Physical age of 64)
P_s	Matches Mortality rates of couples found in Social Security Administration Period Life Tables 2015
Labor endowments	
$\eta_e = 7$	Size of labor endowment grid
$\rho = 0.96$	Persistence of labor endowments
$\sigma_e = 0.212$	Standard deviation of transitory shocks
$\rho_{IG} = 0.41$	Intergenerational persistence of labor endowments
$\sigma_{e_1} = 0.616$	Standard deviation of age 1 endowment shock
$a_{IG} = 19$	Age of intergenerational transmission (physical age 40) taken from Hendricks (2007)
Preferences	
$\sigma = 1.5$	Consistently used throughout the literature
$\rho_j = 0.5$	Intergenerational preference transmission is set to 0.5
Technology	
$\alpha = 0.36$	Capital income share
$\delta_k = 0.076$	Jointly set with A to normalize $q_L = 1$ when $r = 0.04$
$A = 0.89$	Jointly set with δ_k to normalize $w = 1$ when $r = 0.04$
Government	
$\tau_w = 0.40$	Tax rate on labor income, Trostel(1993), Hendricks (2007)
$\tau_R = 0.4 * (AvgEarnings)$	Retirement transfer set to 40% of average household earnings

The parameters listed above are common to all model specifications. In the results section I distinguish experiments by the proportion of agents endowed with each β_j and δ_j.

4.2 Labor endowments

Agent's labor endowments consist of a deterministic age efficiency profile, $h(a)$, and a stochastic labor productivity shock, e. The age efficiency profile is taken from Huggett 1996. The transition matrix for labor endowment shocks, P_e, is the Markov approximation of the following autoregressive process:

$$\ln(e_t) = \rho \ln(e_{t-1}) + \epsilon_t \tag{6}$$

where e_{t-1} is the labor shock experienced in the previous period and $\epsilon_t \sim N(0, \sigma_\epsilon^2)\ \forall\ t$.

[10] A weighted average of Male and Female survival probabilities is used for model calibration. All values are taken from the 2015 Social Security Life Tables

This provides a grid of potential labor shocks and a Markov transition matrix describing the probability of moving from shock e_i to any other shock e_j, $j \in [1, 7]$.

The initial productivity shock of young agents, e_1, is equal to their parent's shock at age a_{IG} with probability 0.41 and is drawn from a normal distribution governing initial labor endowments with probability 0.59[11]. Initial labor endowments are distributed $N \sim (\bar{e}_1, \sigma_1^2)$.

4.3 Preferences

Preferences over consumption utility are given by $u(c) = c^{(1-\sigma)}/(1-\sigma)$. The curvature parameter of the household utility function is set to $\sigma = 1.5$. This is the value chosen in Hendricks (2007), Huggett (1996), and DeNardi and Yang (2014), among others.

In terms of the calibration of present bias, Laibson, Repetto and Tobacman (2007) argue for a degree of present bias equal to 0.7, while Paserman (2008) finds an average degree of present bias of 0.65. Meier and Sprenger (2015) utilize a series of survey questions and find an average degree of present bias to be between 0.69 and 0.82. Tanaka et al. (2010) perform a similar survey based experiment in rural Vietnam and find an average degree of present bias of 0.644. Due to the disparate findings outlined above in the empirical literature, I calibrate present bias as 0.7 in each experiment performed in Section 6.[12]

4.4 Technology

The production function in the model economy is of the Cobb-Douglas form. $F(K, L) = AK^{\alpha}L^{(1-\alpha)}$. The capital share, α, is set equal to 0.36. A and δ are chosen so that $w = 1$ when the interest rate, r, equals 0.04 and the capital to output ratio, $\frac{K}{Y}$, equals 3.1.

4.5 Government

Wages are taxed at a rate of 0.40, following Huggett (1996)[13]. Intergenerational transfers are not taxed in the baseline case ($\tau_B = 0$) following a convention in much of the quantitative life-cycle literature. Preliminary analysis indicates my results are not sensitive to assumptions regarding inheritance taxation.

5 Overview of Experiments

In this section, I draw distinctions between several experiments regarding the distribution of societal preferences over β and δ. As there is no consensus in the literature regarding the true distribution of household preferences, I remain agnostic with respect to which experiment corresponds to the "correct" model[14]. To impose discipline across modeling experiments, the aggregate capital to output ratio is set so that $\frac{K}{Y}= 3.1$ in each model economy and the interest rate is set so that $r = 4\%$. Each experiment is disciplined by the selection of a common discount factor β_c applied to every household in the economy so that equilibrium

[11] I follow Hendricks (2007) in setting $\rho_{IG} = 0.41$ and setting $a_{IG} = 39$.

[12] The results reported in Section 6, while quantitatively sensitive to the selection of δ do not change qualitatively when alternate parameterizations are considered.

[13] Note, the results presented in the next section are robust to changes in the tax rate.

[14] See Frederick, Loewenstein and O'Donogue (2002) for a discussion of the empirical estimates of β dispersion. Their results indicate that low end estimates of β can be close to 0 and high end estimates can be greater than 1.

r and $\frac{K}{Y}$ match the targets outlined above.[15]. The term "Avg. β" refers to the average exponential discount factor across households in the model economy.

It is important to note that rather than iterating over β_c to match a targeted r, one could just as easily iterate over r for a fixed value of β_c to find an equilibrium. I avoid this approach as (1) fixing r constitutes holding an observable variable fixed while varying an unobservable variable (β_c) and (2) changing r changes the price of renting capital and creates savers and spenders in a model with heterogeneous preferences. Changing the value of β_c does *the exact same things*. With heterogeneous preferences and a fixed r, a high β_C will generate individuals who view this interest rate as too high[16] and therefore become savers while others will view the interest rate as too low and become spenders. A change in β_c can thus be interpreted as a change in the *effective interest rate,* $\beta_c \times r$.

The experiments considered in the following section are outlined in Table 2:

Table 2: Outline of Experiments

	Proportion of Households by Type						
	β			δ			
Experiment	(0.98	0.94	0.90)	(1.0	0.7)	β_c	Avg. β
Base	1.0	-	-	1.0	-	0.983	0.963
BasePB	1.0	-	-	-	1.0	1.017	0.999
BaseHet	0.5	0.5	-	1.0	-	0.997	0.957
	β			δ			
Experiment	(0.98	0.96	0.90)	(1.0	0.7)	β_c	Avg. β
Full	.44	.12	.44	1.0	-	1.003	0.945
FullPB	.44	.12	.44	-	1.0	1.027	0.968
*FullPBHet$_\lambda$	.44	.12	.44	λ	$1-\lambda$	-	-

where the "Proportion of Households by Type" represents the stable distribution of households over each potential discount factor in the model economy.

The experiment labeled "Base" corresponds to a baseline model in which all agents are exponential discounters ($\delta = 1.0$) and have the same $\beta = 0.963$ ($0.98 \times \beta_c = 0.98 \times 0.983$) in equilibrium. "BasePB" refers to a model in which all agents are present-biased ($\delta = 0.7$) and have an exponential discount factor equal to 0.999 in equilibrium. "BaseHet" refers to a model in which all agents are exponential discounters and half of all agents have an equilibrium discount factor of 0.977 (0.98×0.997) and the other half have an equilibrium discount factor of 0.937 (0.94×0.997).

All experiments labeled "Full" refer to a richer model in which agents are heterogeneous in their exponential discount factor β. Calibration for preference heterogeneity is modeled after Hendricks (2007). "Full" refers to a model in which agents are all exponential discounters ($\delta = 1.0$) with heterogeneous exponential discount factors. "FullPB" refers to a model with

[15] More detailed information regarding the solution algorithm can be found in Appendix C.

[16] In relation to the value of β that would lead to consumption smoothing in their consumption Euler equation.

the same heterogeneity in exponential discount factors and $\delta = 0.7$ for all agents. Finally, "FullPBHet" refers to a set of calibrations in which households preferences are distributed over β as described in "Full", but a proportion λ of agents endowed with each β are not present-biased ($\delta = 1.0$) and $(1 - \lambda)$ are present-biased ($\delta = 0.7$).

6 Results

6.1. Baseline Results

Table 3 outlines the Gini coefficient and selected elements of the Lorenz Curve for both US data (SCF and PSID data) and for the "Base" model economies outlined above.

Table 3: Wealth Distribution in the U.S. and "Hugg" Model Economies

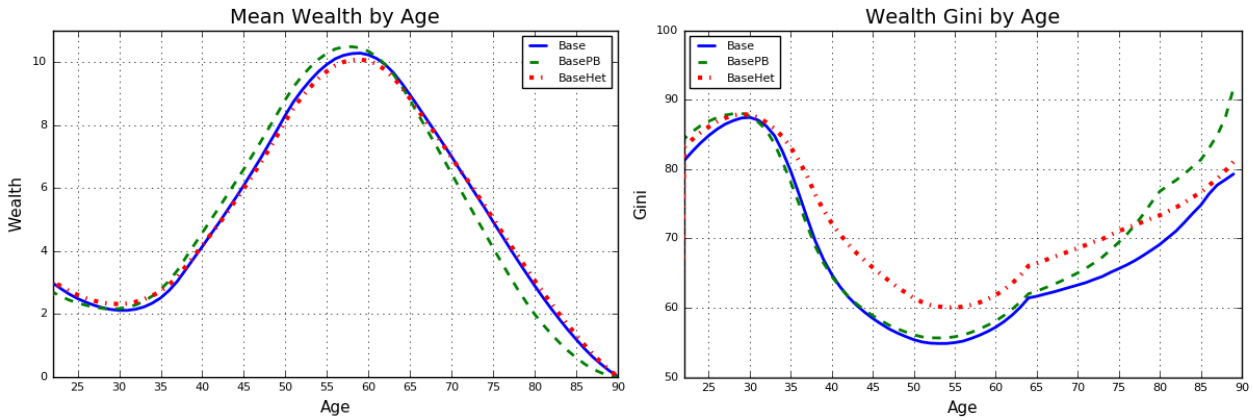
	Gini	99-100	95-99	90-95	80-90	40-80	0-40
PSID (2003)	0.76	25.3	21.8	14.0	16.3	21.8	0.9
SCF (1998)	0.80	34.7	23.1	11.3	12.7	17.2	1.0
Base	0.70	12.0	21.8	16.8	21.8	25.9	1.6
BasePB	0.72	11.8	22.4	17.4	22.3	25.3	0.7
BaseHet	0.74	13.1	24.0	18.3	20.2	23.6	0.8

As shown in Huggett (1996), a model economy with homogeneous exponential discounters generates a Gini coefficient that is lower than that found in the data, with far less right tail wealth than US data indicates. Relative to the baseline model, adding present bias increases the Gini coefficient in the model economy slightly, however both the poorest 40% and the richest 1% of households are too poor relative to the data *and* the baseline model. As shown in Krussell-Smith (1998) and Hendricks (2007), the inclusion of a modest degree of heterogeneity in the exponential discount factor leads to a marked improvement in the Gini coefficient (0.74 compared to 0.70 in the baseline model) and the model's ability to match the wealth holdings of the top 1% of earners (13.1% compared with 12%).

To better understand the distinct role of present bias relative to that of preference heterogeneity in generating equilibrium wealth dispersion, consider the graphs in Figure 2 representing the average wealth accumulated in the model economy by age and the Gini coefficient in the model economy by age across each experiment.

Average wealth in the "Base" model with homogeneous exponential discounters is nearly indistinguishable from the average wealth accumulated in the "BaseHet" model with heterogeneous exponential discounters. However, in the "BasePB" model economy, in which all agents are homogeneous in their exponential discount factor and present-biased, mean wealth peaks at a slightly earlier age and a slightly higher value than in the non present-biased economies. Further, wealth is depleted at a faster rate when all agents are present-biased following peak earnings, particularly after households retire (age 65).

Panel (b) sheds further light on the distinct role played by present bias over the life-cycle. Heterogeneity in discount rates leads to a level shift in the wealth Gini at every age over the life-cycle compared to a model with homogeneous discounters. Present bias, on the



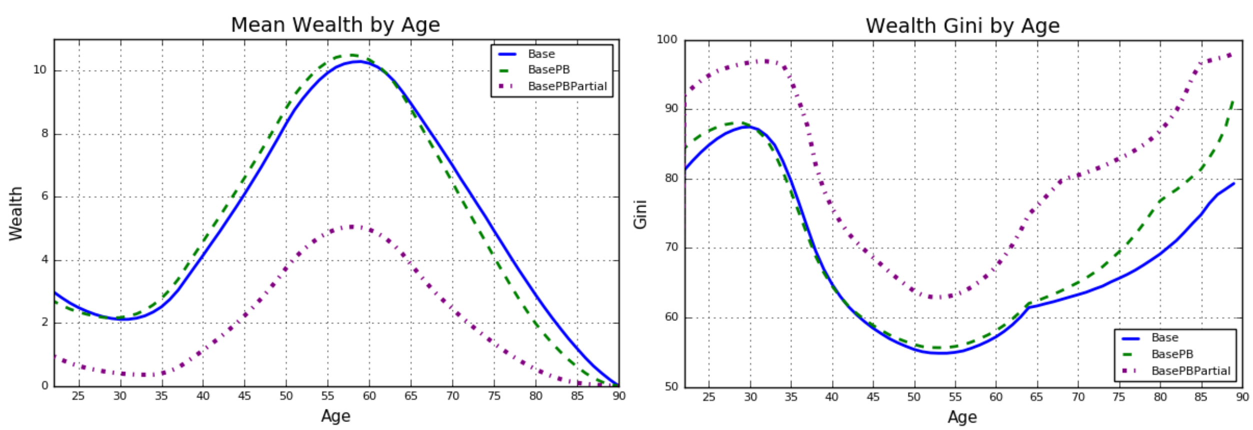
(a) Mean Wealth by Age

(b) Gini Coefficient by Age

Figure 2: Evolution of Wealth over the Life-cycle

other hand, does very little to impact wealth inequality over the life-cycle *until households near retirement*. At retirement, the "BaseHet" economy and the homogeneous exponential discounter economy see a slight uptick in wealth inequality, but wealth inequality in the present-biased economy increases at a much faster rate to a much higher level than either of the exponential economies.

Early in the life-cycle, an economy comprised of present-biased optimizers looks very similar to one comprised of homogeneous discounters, as general equilibrium effects impose a higher effective interest rate ($\beta_c \times r$) in the present-biased society than the "Base" society. As all present-biased agents have a lower preference for savings than rational agents, the relative value of savings increases and overwhelms present-biased agents' desire to over consume while young. Models that do not place present-biased agents in a general equilibrium framework will surely miss this fact. To this point, consider a version of the "BasePB" model that is *not* in general equilibrium, called "BasePB$_{partial}$". Every agent is endowed with $\beta_c = 0.963$ as in the "Base" economy and the interest rate is still set to $r = 4\%$.



(a) Mean Wealth by Age

(b) Gini Coefficient by Age

Figure 3: Wealth Evolution: General vs Partial Equilibrium

The resulting economy has a Gini coefficient of 0.83 and the top 1% of households hold over 19% of all wealth. These are both significant improvements over the baseline model's ability to match inequality in the data, and may lead one to believe that homogeneous present bias plays a tremendous role in generating wealth inequality. Figure 3 shows a model analyzing the role of present bias outside of general equilibrium will drastically misrepresent the role played by present bias, particularly with regard to any measure that depends on aggregate wealth or the interest rate.

In general equilibrium models, the largest deviation between present-biased and non-present-biased societies comes late in life as households with low wealth (highest effective discount rate) are far more tempted by their shortened planning horizon than high wealth present-biased households. As there is already some degree of wealth inequality at retirement across all model economies due to heterogeneity in labor earnings, inequality is amplified in a present-biased society when the shortened planning horizon of old age is interacted with the high temptation of short-term preferences. Just as partial equilibrium models will overstate the importance of present bias in generating wealth inequality, infinite horizon models will understate the role of present bias *even in general equilibrium* as the largest impact of present bias on household outcomes occurs as households reach retirement. Thus, models missing the richness associated with life-cycle dynamics will understate the role of present bias.

6.2. Full Model Results

Results from the "Full" experiments (outlined in Table 2) are presented in Table 4. In each experiment, I vary the percentage of the population with present-biased preferences, where a proportion λ of the agents in the model are not present-biased ($\delta = 1.0$) and $(1 - \lambda)$ of the agents are present-biased ($\delta = 0.7$). The results from my calibration of the "Base" model as well as the wealth moments in PSID and SCF data are reported in Table 4 for ease of comparison.

Table 4: Wealth Distribution in the U.S. and "Full" Model Economies

			Gini	99-100	95-99	90-95	80-90	40-80	0-40
PSID (2003)			0.76	25.3	21.8	14.0	16.3	21.8	0.9
SCF (1998)			0.80	34.7	23.1	11.3	12.7	17.2	1.0
	λ	β_c							
Base	1	0.983	0.70	12.0	21.8	16.8	21.8	25.9	1.6
Full	1	1.003	0.77	13.8	26.3	19.6	20.6	19.7	0.1
FullPBHet$_{.75}$	3/4	1.008	0.78	14.0	27.2	19.6	20.7	18.5	0.0
FullPBHet$_{.5}$	1/2	1.013	0.79	14.2	27.9	19.6	20.8	17.5	0.0
FullPBHet$_{.25}$	1/4	1.020	0.80	14.4	27.9	19.9	20.7	17.1	0.0
FullPB	0	1.027	0.79	13.6	27.5	20.2	21.0	17.6	0.0

As shown in Hendricks (2007), the "Full" model in which there is discount rate heterogeneity and no present bias offers a significant improvement in fit relative to the "Base" model. The Gini coefficient on wealth is 0.77 (compared to 0.70 in the "Base" model and

0.76-0.80 in the data) and the percentage of wealth held by 99^{th} percentile of households is 13.9. Augmenting the baseline "Full" model so that every individual is present-biased ("FullPB") results in a slightly higher Gini coefficient of 0.79 but the percentage of wealth held by the top 1% of households is reduced relative to a model with no present bias. Again, we see the powerful role played by general equilibrium as the negative impact of agents' biases are somewhat mitigated by the increased relative prices of capital.

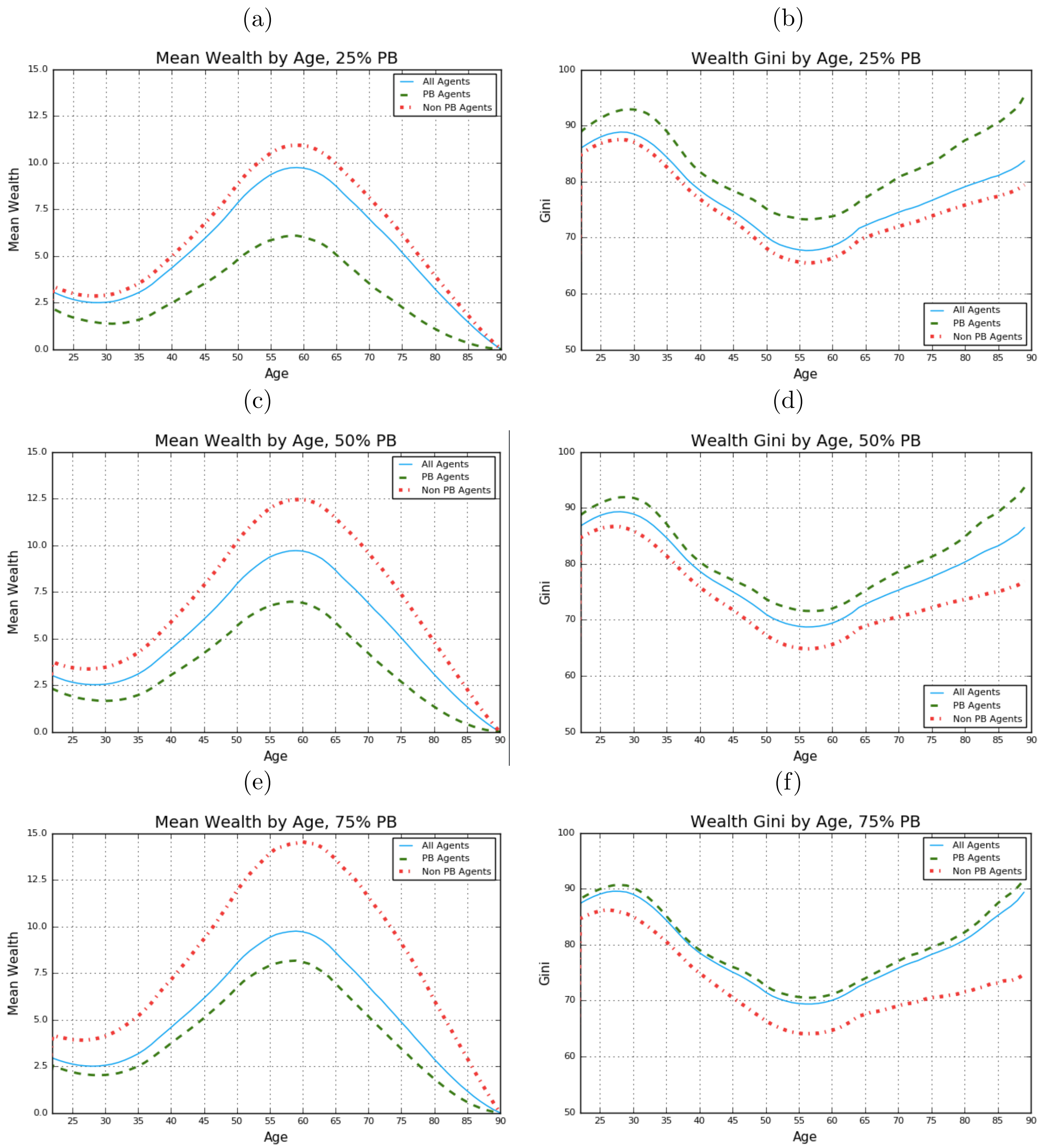


Figure 4: Evolution of Wealth over the life-cycle

As I vary the proportion of individuals who are present-biased from 0 to 1, there is a steady increase in both the Gini coefficient **and** the percentage of wealth held by the 99^{th} percentile of earners. The model in which 75% of households are present-biased ("FullPB$_{.25}$") results in a wealth share for the top 1% of earners of 14.4, which is 20% higher than the equivalent wealth share in the "Base" calibration and 4.5% higher than the wealth share in the "Full" calibration in which no agents are present-biased.

The rationale behind this finding is fairly straightforward. If we fail to embed present-biased agents in a general equilibrium model in which prices respond to their actions, the resulting model economy will overstate the role of present bias in generating wealth inequality. When agents are heterogeneous with regard to their present bias, present-biased agents are essentially embedded in a partial equilibrium model, albeit one in which the price (or in this case, the value of β_c) is closer to their desired price than it would be if every agent in the economy was not present-biased.

Figure 4 provides further insight into the evolution of wealth inequality in each "FullPB-Het" calibration. The dashed line represents the mean wealth (or wealth Gini) by age of present-biased agents for $\lambda = 0.75$, 0.50, and 0.25. The dash-dot line represents these same statistics for the non present-biased agents in each experiment, and the solid line represents all agents in the model economy.

Comparing panels (a), (c), and (e), it is apparent that as the proportion of individuals in the model economy endowed with present bias increases, the gap in mean wealth holdings across agent types increases. Agents in the exponential sub-economy take advantage of the increased effective interest rate they face when coexisting in an economy with a large number of present-biased individuals and amass much higher average wealth than their present-biased peers. Thus, as the percentage of individuals in the economy endowed with present-biased preferences increases, inequality between time-inconsistent and time-consistent agents increases.

A final note of interest from Figure 4 pertains to the exact shape of the wealth Gini by age for all agents in the model economy (the solid line in all three experiments). In panels (b), (d), and (f) this line reaches its minimum between age 55 and 60, but the rate at which inequality evolves following its trough depends on the percentage of agents in the model economy who are present-biased. In panel (b), when only 25% of agents are present-biased, the wealth Gini by age increases at a much slower rate following age 60 to a lower overall level than it does in panel (d) or panel (f). Thus, my model offers a convenient means of backing out the percentage of households who display present-biased preferences via the examination of the evolution of wealth inequality as agents age.

7 Conclusion

I embed present-biased agents into a standard quantitative life-cycle model. If all agents are present-biased, the distribution of wealth is largely unaffected relative to a baseline economy in which all agents are exponential discounters. However, stark differences arise between a present-biased society and a society of homogeneous, exponential discounters when the accumulation of wealth over the life-cycle is analyzed. A present-biased society is characterized by an earlier peak in mean wealth by age and by increased dispersion in wealth as household reach retirement.

I then consider agents who are heterogeneous across both their exponential and present-biased discount factor. I find the inclusion of some present-biased households improves the fit of the model economy to the data. The increase in wealth dispersion resulting from an increase in the percentage of households that are present-biased arises as time-consistent discounters amass higher wealth relative to their present-biased peers due to the general equilibrium impact of present-biased optimization on the effective interest rate. Further, economy-wide old age inequality is driven by the proportion of agents with present-biased preferences. This finding compliments recent work by Schreiber and Weber (2016) and Goda et al. (2019) who find strong support for an empirical relationship between time-inconsistency and poor retirement wealth decisions.

This result has the potential to resolve an issue discussed in Hendricks (2007) in which the distribution of households discount factors is selected to minimize the distance between model Gini coefficients on wealth by age and moments in U.S. data. Hendricks notes "since preference heterogeneity increases inequality among young and old households, it is not possible to match inequality among the old without overstating inequality among the young". As shown throughout Section 6, present bias offers an avenue through which old age wealth inequality can be increased without overstating inequality among younger households. A future endeavor aimed at calibrating the distribution of households over exponential and present-biased discount factors may offer a resolution to this issue while simultaneously shedding light on the percentage of households endowed with present-biased preferences.

Appendix A. Equilibrium

A stationary, competitive equilibrium consists of aggregate quantities ($K,\ L,\ C,\ T,\ B$), prices ($w,\ r,\ q_L,\ q_K$), transfers ($\tau(x)$), current and continuation value functions ($V(x)$ and $\tilde{V}(x)$), policy functions ($c(x)$ and $k(x)$) and a distribution over agent types ($\Lambda(x)$) such that:

- The policy functions ($c(x)$ and $k(x)$) along with the value function $V(x)$ and the continuation value function $\tilde{V}(x)$ solve the agent's optimization problem.
- Firms maximize profits.
- The government balances its budget.
- The distribution of households over states, $\Lambda(x)$, is stationary.
- Prices are given by $w = (1 - \tau_w)q_L$ and $r = q_K - \delta$.
- Markets Clear:
  (i) $K = \int \Lambda(x)k(x)dx$
  (ii) $L = \int \Lambda(x)l(x)dx$
  (iii) $F(K, L) = C + \delta K$ where $C = \int_x \Lambda(x)c(x)dx$

Appendix B. The Effective Discount Rate

If the Bellman Equation outlined by equations (1)-(3) has an interior solution, such a solution satisfies the present-biased Euler Equation:

$$u'(c_t) \geq \beta s_{t+1} E_t\{u'(c_{t+1})[1 + r - (1-\delta)c_k(k_{t+1}, e_{t+1}, t+1)]\} \tag{7}$$

where u' is the derivative of the utility function and c_k represents the the derivative of the optimal consumption function w.r.t assets. Equation (7) can be re-written in the following way:

$$\begin{aligned}
u'(c_t) &\geq \beta s_{t+1} E_t\{u'(c_{t+1})[1 + r - (1-\delta)c_k(k_{t+1}, e_{t+1}, t+1)]\} \\
\Leftrightarrow u'(c_t) &\geq \beta s_{t+1} E_t\{u'(c_{t+1})\}(1+r) - \beta s_{t+1} E_t\{u'(c_{t+1})(1-\delta)c_k(k_{t+1}, e_{t+1}, t+1)\} \\
\Leftrightarrow u'(c_t) &\geq \beta(1+r)s_{t+1}E_t\{u'(c_{t+1})\}\left[1 - \frac{1-\delta}{1+r}E_t\{c_k(k_{t+1}, e_{t+1}, t+1)\}\right] \\
\Leftrightarrow u'(c_t) &\geq \beta(1+r)s_{t+1}E_t\{u'(c_{t+1})\}\left[1 - \frac{1-\delta}{1+r}\frac{E_t\{u'(c_{t+1})c_k(k_{t+1}, e_{t+1}, t+1)\}}{E_t\{u'(c_{t+1}\}}\right] \\
\Leftrightarrow u'(c_t) &\geq \beta\left[1 - \frac{1-\delta}{1+r}\frac{E_t\{u'(c_{t+1})c_k(k_{t+1}, e_{t+1}, t+1)\}}{E_t\{u'(c_{t+1}\}}\right](1+r)s_{t+1}E_t\{u'(c_{t+1})\}
\end{aligned}$$

let $\beta_{x'} = \beta\left[1 - \frac{1-\delta}{1+r}\frac{E_t\{u'(c_{t+1})c_k(k_{t+1}, e_{t+1}, t+1)\}}{E_t\{u'(c_{t+1}\}}\right]$, then the present-biased Euler Equation can be written as:

$$u'(c_t) \geq \beta_{x'}(1+r)s_{t+1}E_t\{u'(c_{t+1})\}$$

which is exactly equation (5) in the text.

Appendix C. Computational Algorithm

Outline of Equilibrium Solution Algorithm:

1. Propose a candidate interest rate, r, and common discount factor, β_c. For a given β_c:
2. Solve the household problem (find $c(x)$ and $k(x)$) given prices r and w that solve the firm optimization problem for the capital stock implied by the interest rate r.
3. Using the optimized capital decision rule, $k(x)$, compute individual savings decisions for the stable distribution of households.
4. Compute aggregate capital, K_1, the capital stock in the model economy given preferences β_c and the interest rate r. Compute the implied capital to output ratio, K_1/Y_1.
5. If K_1/Y_1 is sufficiently close to the target capital output ratio of 3.10, stop. If not, propose a new β_c and repeat steps (a)-(d) until convergence is achieved.

References

Angeletos, G.M., Laibson, D., Repetto, A., Tobacman, J., and S. Weinberg (2001). The Hyperbolic Consumption Model: Calibration, Simulation, and Empirical Observation. *Journal of Economic Perspectives, vol. 15 no. 3*, 47-68.

DellaVigna, S. (2009). Psychology and Economics: Evidence from the Field. *Journal of Economic Literature, Vol. 47 (2)* 315-372.

DeNardi, M. (2004). Wealth Inequality and Intergenerational Links. *Review of Economic Studies 71(3)*, 743-768.

DeNardi, M. and F. Yang (2014). Bequests and Heterogeneity in Retirement Wealth. *European Economic Review 72*, 182-196.

Frederick, S., Loewenstein, G., and T. O'Donoghue (2002). Time Discounting and Time Preference: A Critical Review. *Journal of Economic Literature, Vol. 40 (2)*. 351-401.

Goda, G., Levy, M., Manchester, C.F., Sojourner, A., and J. Tasoff (2019). Predicting Retirement Savings Using Survey Measures of Exponential-Growth Bias and Present Bias. *Economic Inquiry, 57(3),* 1636-1658.

Harris, C. and D. Laibson (2001). Dynamic Choices of Hyperbolic Consumers. *Econometrica, 69(4),* 935-957.

Hendricks, L. (2007). How Important is Discount Rate Heterogeneity for Wealth Inequality? *Journal of Economic Dynamics and Control, 31(9),* 3042-3068.

Huggett, M. (1996). Wealth Distribution in Life-Cycle Economies. *Journal of Monetary Economics, 38*, 469-494.

İmrohoroğlu, A., İmrohoroğlu, S, and D.H. Joines. (2003). Time-inconsistent preferences and social security. *Quarterly Journal of Economics*, 745-784.

Krusell, P. and A.A. Smith (1998). Income and Wealth Heterogeneity in the Macroeconomy. *Journal of Political Economy 106,* 868-896.

Krusell, P., Kuruscu, B., and A.A. Smith (2002). Equilibrium Welfare and Government Policy with Quasi-geometric Discounting. *Journal of Economic Theory, 105,* 42-72.

Laibson, D. (1997). Golden eggs and hyperbolic discounting. *The Quarterly Journal of Economics- Volume 112, Issue 2*, 443-477.

Laibson, D. (2015). Why don'e present-biased agents make commitments? *American Economic Review Papers and Proceedings; 105(5)*, 267-272.

Laibson, D., Maxted, P., and B. Moll (2020). Present Bias Amplifies the Household Balance-Sheet Channels of Macroeconomic Policy. *Working Paper.*

Maliar, L. and S. Maliar (2006). The Neoclassical Growth Model with Heterogeneous Quasi-Geometric Consumers. *Journal of Money, Credit, and Banking 38(3),* 635-654.

Meier, S. and C.D. Sprenger (2015). Temporal Stability of Time Preferences. *The Review of Economics and Statistics, 97(2),* 273-286.

Paserman, M.D. (2008). Job Search and Hyperbolic Discounting: Structural Estimation and Policy Evaluation. *The Economic Journal, 118,* 1418-1452.

Phelps, E.S., and R.A. Pollak (1968). On Second-Best National Saving and Game Equilibrium Growth. *Review of Economic Studies, XXXV*, 185-199.

Schreiber, P. and M. Weber (2016). Time Inconsistent Preferences and the Annuitization Decision. *Journal of Economic Behavior & Organization, volume 129*, 37-55.

Strotz, R. H. (1956). Myopia and Inconsistency in Dynamic Utility Maximization. *Review of Economic Studies, XXIII*, 165-180.

Tanaka, T. Camerer, C.F., and Q. Nguyen (2010). Risk and Time Preference: Linking Experimental and Household Survey Data from Vietnam. *The American Economic Review, Volume 11 No. 1*, 557-571.

Trostel, P. (1993). The Effect of Taxation on Human Capital. *Journal of Political Economy, 101(2),* 327-350.